Assignment 5

Q1) C

Q2) B

Q3) C

Q4) C

Q5) $\sin^2 60^\circ \sec 60^\circ - \operatorname{cosec} 60^\circ = \frac{9-4\sqrt{3}}{6}$

$\sin^2 60^\circ = \frac{\sqrt{3}}{2}$

$\sec 60^\circ = 2$

$\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$

$$\frac{\sqrt{3}}{2} \times 2 - \frac{2}{\sqrt{3}} = \frac{9-4\sqrt{3}}{6}$$

* $2 \times \sqrt{3} = 6$

* $2 \times 2 = 4 \quad = 4\sqrt{3}$

* $3 \times 3 = 9$

Q6)

a) $(\sec^2 B - 1)(\cot^2 B + 1) = \sec^2 B$

$-1 + 1 = 0$ (cancel each other out)

$\sec^2 B = \sec\theta$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot^2 B = \cot\theta$

$\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{c}{a}$

$\cot\theta = \frac{a}{b}$

$\frac{c}{a} \times \frac{a}{b} = \frac{\cancel{a}c}{\cancel{a}b}$

$= \frac{c}{b}$

$= \sec(90 - \theta)$

$\therefore = \sec^2 B$

Q6)

b) $\sin(90-\theta)\cot(90-\theta) = \sin\theta$

$\sin\theta = \sin(90-\theta)\cot(90-\theta)$

$\sin\theta = \cos\theta \times \tan\theta$

$= \frac{a}{c} \times \frac{b}{a} = \frac{\cancel{a}b}{\cancel{a}c}$

$= \frac{b}{c}$

$= \sin\theta$

c) $2\tan A \sec A = \frac{\cancel{A}\sin A}{1-\sin A} + \frac{\sin A}{1+\sin A}$

$= \frac{1\sin A + 1\sin A}{\cancel{(1-\sin A)}\cancel{(1+\sin A)}} = 2\tan A$

$1 + \tan^2\theta = \sec^2\theta$

$= \frac{2\tan A}{(1-\cancel{\sin A})(1+\cancel{\sin A})}$

$1 \times 1 = 1$

$= \frac{2\tan A}{1}$

$= 1\tan A = \sec A$

$\therefore \cancel{=} 2\tan A \sec A$

Q7)

a) $\cos 2\theta = \frac{1}{2}$ $[-90°, 90°]$

Cos is > 0 in the 1st and 4th quadrant.

$\cos 2\theta = 90°, 360° - 90°$
$= 90°, 270°$

b) $4\sin^2\theta = 3$ $[0, 2\pi]$

$\sin^2\theta$ is > 0 in the 1st and 2nd quadrant.

$4\sin^2\theta = 3, 2\pi - 3$
$= 3, 3.28$

c) $(\csc\theta - 1)\left(\sec\theta + \frac{2}{\sqrt{3}}\right) = 0$ $[-180°, 180°]$

$\left.\begin{array}{l}\csc\theta - 1 = \sec\theta \\ \sec\theta = \csc\theta\end{array}\right] =$ Complementary angle

$0 - 180° = -180°$
$180° - 0 = 180°$

Q8) $f(x) = \dfrac{x}{\sqrt{a^2 - x^2}}$

$$f(a\sin\theta) = \frac{a\sin\theta}{\sqrt{a - a\sin\theta}}$$

$$= \frac{a\sin\theta}{a(a - \sin\theta)}$$

Q9) $y = 1 + \cancel{\cos} 3\cos(2x)$

x	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{3\pi}{3}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{6\pi}{3}$	$\frac{7\pi}{3}$	$\frac{8\pi}{3}$	$\frac{9\pi}{3}$	$\frac{10\pi}{3}$
Cos 2 (y)	1.05	2.09	3.14 (π)	4.19	5.23	6.28	7.33	8.37	9.42	10.47

amplitude $= 3$

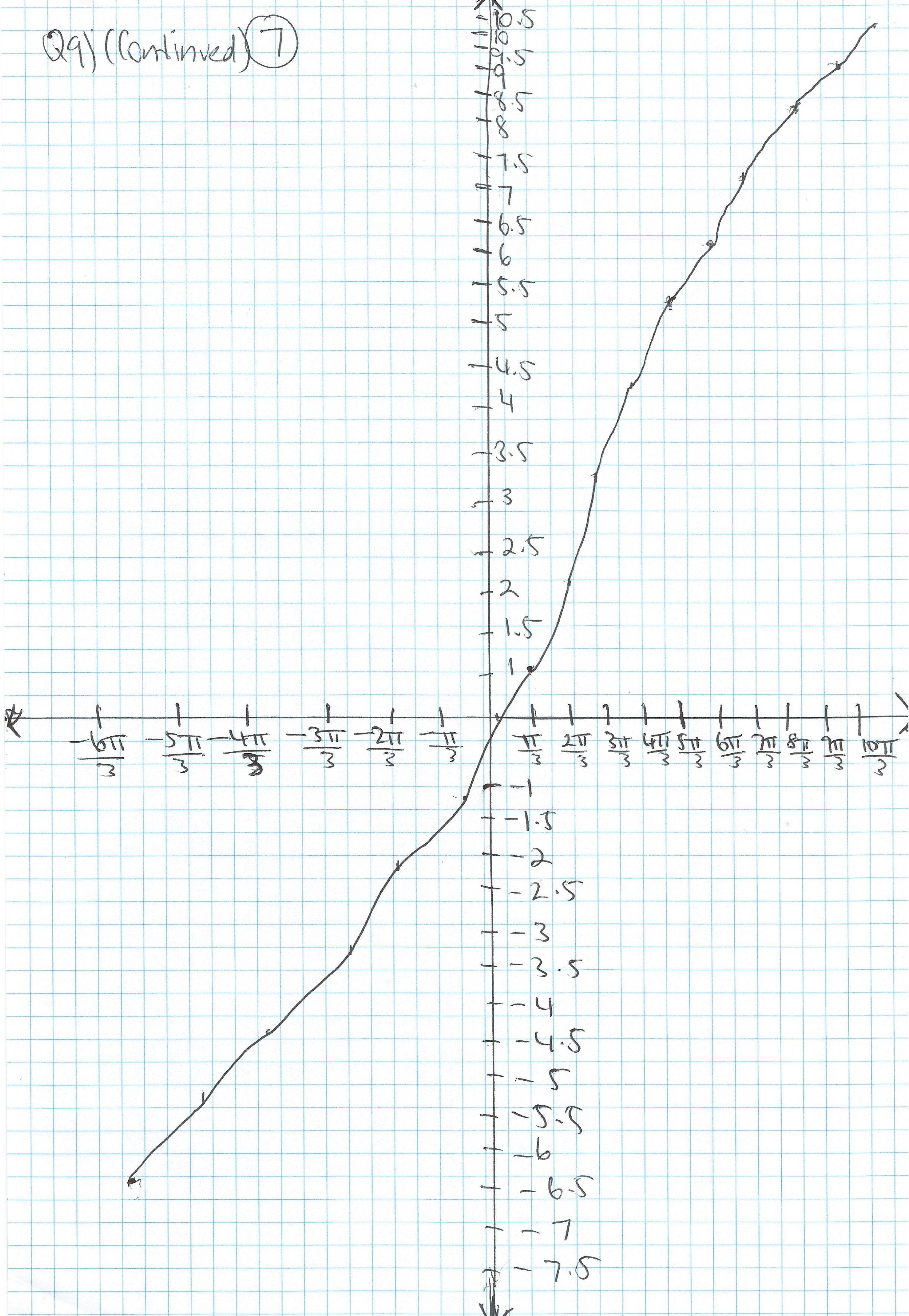
Period $= \frac{3\pi}{3}$

Centre $= 1$

Range $= [1, 11]$

Sketch/graph is on the next page $\rightarrow$

Q9) (Continued) 7



Q10)

a)

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b) $(1\times\frac{1}{6})+(2\times\frac{1}{6})+(3\times\frac{1}{6})+(4\times\frac{1}{6})+$
$(5\times\frac{1}{6})+(6\times\frac{1}{6}) = 3.5$

c) $(1-3.5)^2\frac{1}{6}+(2-3.5)^2\frac{1}{6}+(3-3.5)^2\frac{1}{6}+$
$(4-3.5)^2\frac{1}{6}+(5-3.5)^2\frac{1}{6}+(6-3.5)^2\frac{1}{6}+\frac{1}{6}$

$= 3.083333333$

$= 3.08\dot{3}$

d) $\sigma=\sqrt{3.083333333}$

$\sigma = 1.755942292$

$\sigma = 1.76$

Q11)

a) First determine probability of each die number being rolled.

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Then a table showing cost of game/payout outcomes.

x	1	2	3	4	5	6
$P(X=x)$	-1	0.5	0.5	0.5	0.5	3

Then payout probability:

M($)	\$-1	\$0.5	\$3
$P(M=m)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

b) $E(M) = \left(-1 \times \frac{1}{6}\right) + \left(0.5 \times \frac{2}{3}\right) + \left(3 * \frac{1}{6}\right)$

$= \$0.667$

Q11)

c) Probability of no payout $= \frac{4}{6}$ or $\frac{2}{3}$

Probability of game player losing \$1 $= \frac{1}{6}$

$\therefore P(\text{player loses}) = \frac{4}{6} + \frac{1}{6}$

$= \frac{5}{6}$

Therefore $P(\text{player wins}) = \frac{1}{6}$ and the game organiser is expected to win.

How many games would need to be played to make \$20/hour?

↓

Expected value of 1 game = \$0.667

$20 \div 0.667 = 30$

$\therefore$ 30 games would have to be played.