Assignment 4

Q1) D

Q2) D

Q3) C

Q4) D

Q5) A

Q6)

4 black balls + 3 white balls = 7 balls

a) Probability of first ball being white is $\frac{3}{7}$

$$P(W_1) = \frac{3}{7}$$

After taking the first ball out, there are six balls left: 4 black and 2 white

$$P(W_2) = \frac{2}{6} = \frac{1}{3}$$

Q6)
a) (Continued)

Probability of both balls being white

$$P(W_1 \cap W_2) = \frac{3}{7} \times \frac{1}{3}$$

$$= \frac{3}{21}$$

$$= \frac{1}{7}$$

b) Probability of first ball being white is $\frac{3}{7}$

$$P(W) = \frac{3}{7}$$

After the first ball is drawn, 6 balls remain: 4 black and 2 white.

$$P(B) = \frac{4}{6} = \frac{2}{3}$$

$$P(W \cap B) = \frac{3}{7} \times \frac{2}{3}$$

$$= \frac{6}{21} = \frac{2}{7}$$

Q6)

c) Probability of both balls being white was proven in (a) to be $\frac{1}{7}$

$$P(\text{Both White}) = \frac{1}{7}$$

Probability of first ball being black is $\frac{4}{7}$

$$P(B_1) = \frac{4}{7}$$

After the first ball is drawn, 3 black balls and 3 white balls are left.

$$P(B_2) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{Both Black}) = \frac{4}{7} \times \frac{1}{2}$$

$$= \frac{4}{14} = \frac{2}{7}$$

$$\therefore P(\text{Same Colour}) = \frac{1}{7} \times \frac{2}{7}$$

$$= \frac{2}{49}$$

Q6)

d) The probability of the first ball being black is $\frac{4}{7}$

After the first ball is drawn, 6 balls remain; 3 black, 3 white.

$$P(B_2) = \frac{3}{6}$$

$$= \frac{1}{2}$$

Q7)

a) $1 - \frac{1}{5}$

$$= \frac{4}{5}$$

b) $P(\text{pink flower}) = \frac{1}{5}$

$P(\text{other colour flower}) = \frac{4}{5}$

$P(\text{at least one pink}) = \frac{1}{5} \times \frac{4}{5}$

$$= \frac{4}{25}$$

(27)

c) $P(\text{pink flower}) = \frac{1}{5}$

$0.99 = \frac{99}{100}$

$\frac{99}{100} \times \frac{1}{5} = \frac{99}{500}$

500 seeds (at least) would have to be planted.

(28)

a) $\frac{4}{30} = \frac{2}{15}$

b) For this question "defective" = D

$$P(D_1) = \frac{2}{15}$$

When one globe is picked, 29 remain: 26 non-defective and 3 defective.

$$P(D_2) = \frac{3}{29}$$

$$P(D_1 \cap D_2) = \frac{2}{15} \times \frac{3}{29}$$

$$= \frac{6}{435}$$

$$= \frac{2}{145}$$

c) 30 light globes: 26 non-defective and 4 defective.

For this question "non-defective" = N

$$P(N_1) = \frac{26}{30} = \frac{13}{15}$$

When one globe is picked, 29 remain: 25 non-defective and 4 defective.

Q8)

c) (continued)

$$P(N_2) = \frac{25}{29}$$

$$P(N_1 \cap N_2) = \frac{13}{15} \times \frac{25}{29}$$

$$= \frac{325}{435}$$

$$= \frac{65}{87}$$

d) 30 light globes: 26 non-defective and 4 defective

$$P(D) = \frac{4}{30} = \frac{2}{15}$$

One globe is picked, 29 remain: 26 non-defective and 3 defective.

$$P(N) = \frac{26}{29}$$

$$P(\text{at least one defective}) = \frac{2}{15} \times \frac{26}{29}$$

$$= \frac{52}{435}$$

Q9)

a) $2^{x-3} = 14.9$

$2^{(6.9-3)} = 14.928852786$

$= 14.9$

$x = 6.9$

b) $\log_x 7 = \frac{\log_2 7}{\log_2 5}$

$\log_x 7 = \frac{2.10720997}{1.50514998}$

$\log_x 7 = 1.401$

$x = 1.58$

c) $9^{2x+4} = 1594323$

$x = 0.7267839$

d) $\ln(x+2) = -3$

$x = 18.086$

Q10) $y = \frac{e^{-3x}}{x^3 + 4}$

$x = 0.23$

$y = \frac{e^{(-3 \times 0.23)}}{0.23^3 + 4}$

$y = \frac{1.993715533}{4.012167}$

$y = 0.4969173848$

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{0.4969173848}{0.23}$

$m = 2.16$

Q11)

a) $\log a(PQ)$

$\log a(3.8 \times 7) = 1.424881637$

$= 1.42$

Q11)

b) $\log_a\left(\frac{P}{Q}\right)$

$\log_a\left(\frac{3.8}{7}\right)$

$= -0.265314444434$

$\doteq -0.27$

c) $\log_a(Q^3)$

$= \log_a(7^3)$

$= 2.535294412$

$= 2.54$

Q12)

a) \$100,000

b) $V = 100000e^{0.3t}$

$V = 100000e^{(-0.3\times5)}$

$V = \$22,313.02$

c) $V = 100000e^{-0.3t}$

$V = 100000e^{(-0.3\times2.31)}$

$V = \$50,007.36$

$\doteq 2.31$ years

Q12)

d) Value after 5 years was found in (a) to be $22,313.02.

Original value = $100,000

$$\frac{22313.02}{100,000} \times 100$$

$$= 22.31\%$$